$$\frac{dF_L}{dz} = \frac{du_L}{dz} - \frac{1}{W_L}\frac{dw}{dz} - \frac{1}{W_L}\frac{dq}{dz}$$

For flow in a duct with no heat transfer into the duct and no shaft work done by the flow, conservation of energy leads to the well-known result that the integral in Equation (5)

$$\int_A \rho V\left(\frac{P}{\rho} + u + \frac{V^2}{2g_c} + \frac{g}{g_c} z\right) dA = \text{const} \quad (5)$$

taken over any duct cross section normal to the flow is a constant. Applying Equation (5) directly to the total two-phase flow in the section dz one obtains Equation (6):

$$\frac{d}{dz}\left[\rho_G V_G A_G \left(\frac{P}{\rho_G} + u_G + \frac{V_G^2}{2g_c} + \frac{g}{g_c} z\right) + \rho_L V_L A_L \left(\frac{P}{\rho_L} + u_L + \frac{V_L^2}{2g_c} + \frac{g}{g_c} z\right)\right] = 0 \quad (6)$$

Performing the differentiation, rearranging terms, and dividing through by $V_G A_G + V_L A_L$ one obtains Equation (7):

$$\frac{dP}{dz} + \frac{1}{V_G A_G + V_L A_L}\left[W_G \frac{V_G}{g_c}\frac{dV_G}{dz} + W_L \frac{V_L}{g_c}\frac{dV_L}{dz} + \frac{g}{g_c}(W_G + W_L) + W_G \frac{du_G}{dz} + W_L \frac{du_L}{dz}\right] = 0 \quad (7)$$

This is the correct energy equation for the combined two-phase flow.

Now consider Equations (2) and (3). If Equation (2) is weighted by W_G and Equation (3) is weighted by W_L and the two equations are added, the terms representing the heat transfer and work done between phases cancel out and Equation (7), the correct two-phase flow energy equation is obtained. If however one weights Equation (2) by $\rho_G A_G dz$ and Equation (3) by $\rho_L A_L dz$ and sums, the terms representing the heat transfer and work done between phases do not cancel out and one obtains an equation which, unlike Equation (7), does not have a clear physical interpretation as the energy equation for the total flow.

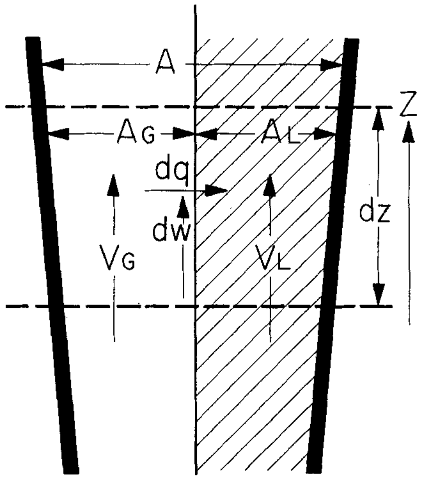

Fig. 1.

An objection raised by the authors of reference *1* to the use of W_G and W_L as weighting factors is that in dt time the gas phase moves a distance $W_G\, dt/\rho_G A_G = V_G\, dt$, whereas the liquid moves $W_L\, dt/\rho_L A_L = V_L\, dt$. Thus the only way for the incremental distance dz to be the same for the gas and liquid phase is for $V_G = V_L$.

Since it is not precisely clear to this present writer how this objection is intended to apply to the differential equations presented here, no attempt will be made to discuss the objection However in view of the demonstrated correctness of Equation (7) as the energy equation for a total two-phase flow, the objection raised by Isbin and Yung Sung Su to the use of W_G and W_L as weighting factors would seem to be not valid.

NOTATION

A = total cross-sectional area of duct, sq. ft.
A_G = cross-sectional area of flow for gas phase, sq. ft.
A_L = cross-sectional area of flow for liquid phase, sq. ft.
$(dF)/(dz)$ = derivative representing rate of change of fluid internal energy plus rate of change of flow energy due to heat transfer and shear work, ft.-lb./lb. mass-ft.
g = gravitational acceleration, ft./hr.^{2}
g_c = gravitational constant, 4.17 × 10^8 ft./hr.^{2}
P = pressure, lb./sq. ft.
t = time, hr.
V = velocity, ft./hr.
W = mass flow rate, lb. mass/hr.
z = vertical height, ft.
ρ = density, lb. mass/cu. ft.
u = thermodynamic internal energy, ft.-lb./lb.mass
dw = incremental rate of work done by gas phase or liquid phase in length dz, ft.-lb./hr.
dq = incremental rate of heat transfer to liquid phase from gas phase in length dz, ft.-lb./hr.

Subscripts

G = gas phase
L = liquid phase

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Use of Momentum and Energy Equations in Two-Phase Flow

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Recently some discussion has arisen as to the proper use of the momentum equation and mechanical energy equation when applied to two-phase flow systems (*5*). A number of publications have appeared in which pressure drop in two-phase fluid flow has been correlated in terms of the total wall shearing stress, while in other work the energy dissipation has been used as the basis of correlation. The purpose of this communication is to present derivations of momentum and mechanical energy equations and to show their relation to pressure-drop correlations in two-phase flow.

Consider first a single fluid phase. The momentum equation at a point can be written as

$$\frac{1}{g_c}\left[\frac{\partial(\rho \mathbf{V})}{\partial\, t} + \nabla\cdot(\rho \mathbf{V}\mathbf{V})\right] = -\nabla p + \nabla\cdot\overline{\overline{\tau}} + \rho \mathbf{F} \quad (1)$$

The mechanical energy equation is obtained from the momentum equation by forming a scalar product with the velocity vector (*8*, *3*):

$$\frac{1}{2g_c}\left[\frac{\partial(\rho V^2)}{\partial\, t} + \nabla\cdot(\rho V^2 \mathbf{V})\right] = -\mathbf{V}\cdot\nabla p + \nabla\cdot(\overline{\overline{\tau}}\cdot\mathbf{V}) - (\overline{\overline{\tau}}\cdot\nabla)\cdot\mathbf{V} + \rho\mathbf{F}\cdot\mathbf{V} \quad (2)$$

For one-dimensional, incompressible, steady flow of a gas Equation (1) can

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INFORMATION RETRIEVAL

Key Words: Diffusion-I, Thermal-I, Separation-I, Mass Transfer-I, Transport-I, Length-F, Separation-G, Heptane-A, Benzene-A, Hydrocarbons-A, Columns (Process)-J, Concentric-J, Annular-J.

Abstract: The phenomenological theory of thermogravitational thermal diffusion columns has been tested in a series of experiments by obtaining data on the steady state separation and approach to equilibrium in a batch column as a function of length. Data were obtained on the separation of an equimolar n-heptane-benzene solution at five different values of column length varying from 14.5 in. to 6 ft. All measurements were made in one concentric cylinder column in which the effective length was varied by controlling the liquid level in the annulus.

Reference: Crownover, Carl F., and John E. Powers, **A.I.Ch.E. Journal, 8,** No. 2, p. 166 (May, 1962).

Key Words: Absorption-H, Mass Transfer-H, Rates-H, Reactions-H, Fluids-I, Gases-I, Liquid Phase-I, Reactions-I, Interfaces-I, Mixtures-I, Carbon Dioxide-A, Oxides (Inorganic)-A, Monoethanolamine-A, Alcohols-A, Amines-A, Water-E, Columns (Process)-J, Wetted Wall-, Predictions-I, Theory-J.

Abstract: Gas absorption accompanied by a fast, second-order chemical reaction has been studied in a wetted-wall column. The observed rates of absorption of pure carbon dioxide in aqueous solutions of monoethanolamine were compared with theoretical predictions. Nonequilibrium effects at the interface were negligible. Absorption from mixed gas streams also was studied, and the results were correlated empirically.

Reference: Emmert, R. E., and R. L. Pigford, **A.I.Ch.E. Journal, 8,** No. 2, p. 171 (May, 1962).

Key Words: Demineralizing-H, Hydrate Process-, Reactions-H, Water-I, Sea Water-I, Hydrates-I, Economics-I, Properties (Characteristics)-F, Costs-G, Properties (Characteristics)-H, Thermodynamic-, Methyl Bromide-I, Brominated Hydrocarbons-I, "Freon" R Fluorinated Hydrocarbons-I, Fluorinated Hydrocarbons-I, Halogenated Hydrocarbons-I.

Abstract: The general attributes of the hydrate process for demineralizing sea water are discussed, and it is shown that the economics of the process depend on the properties of the hydrating agent used. Eight new hydrates are reported, and the thermodynamic properties of three hydrate systems are presented in some detail. These are the hydrates of methyl bromide, Freon 21, and Freon 31.

Reference: Barduhn, A. J., E. E. Towlson, and Y. C. Hu, **A.I.Ch.E. Journal, 8,** No. 2, p. 176 (May, 1962).

Key Words: Rates-H, Reactions-H, Kinetics-H, Dehydration-I, Decomposition-I, Catalysis-I, Alcohols-I, Butanol-A, Propanol-A, Ethanol-A, Alcohols-A, Olefins-B, Hydrocarbons-B, Vinyl Compounds-B, Water-B, Silica-D, Alumina-D, Oxides (Inorganic)-D, Catalysts-J, Beads-J, Pellets-J, Temperature-F, Pressure-F, Rates-G, Constants-G, Models-J, Mechanisms (Chemical)-I, Reaction Mechanisms-I, Diffusion-I, Mass Transfer-I, Porosity-I, Physical Properties-I, Properties (Characteristics)-I.

Abstract: The catalytic dehydrations of n-butanol, n-propanol, and ethanol to produce water and the respective olefins have been studied for the temperature range 400° to 700°F. and over the pressure range 1 atm. to 100 lb./sq. in. gauge. Silica-alumina catalyst beads were employed. The observed rates are explained by a model which postulates a surface reaction mechanism together with intraparticle mass transfer effects.

Reference: Miller, D. N., and R. S. Kirk, **A.I.Ch.E. Journal, 8,** No. 2, p. 183 (May, 1962).

Key Words: Viscometry-I, Measurement-I, Viscosity-H, Properties (Characteristics)-H, Physical Properties-H, Flow-H, Fluid Flow-H, Laminar Flow-H, Annular-H, Fluids-A, Gases-A, Liquids-A, Temperature-F, Pressure-F, Correlations-J, Forecasting-I, Predicting-I, Models-H.

Abstract: A new body with stabilizers removed from the cylindrical section has been designed for falling-cylinder viscometers. Gases and liquids at varying temperatures and pressures have been used to obtain experimental data to evaluate the new body. Experimental results are compared with those predicted by a proposed model. Fluid flow in the annulus remained laminar throughout the study. The results of the investigation allow an improved prediction of the falling-cylinder viscometer constant.

Reference: Lohrenz, John, and Fred Kurata, **A.I.Ch.E. Journal, 8,** No. 2, p. 190 (May, 1962).

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be integrated in the direction normal to flow:

$$\frac{d\,(\overline{W_G V_G})}{g_c} = -A_G dp + (\tau_{LG}\, C_{LG} + \tau_{WG}\, C_{WG})\, dz - \rho_G A_G \frac{g_L}{g_c} dz \qquad (3)$$

Similarly integrating Equation (2) one gets

$$\frac{d\,(\overline{W_G V_G^{\,2}})}{2\,g_c} = -A_G \overline{V}_G\, dp + (\tau_{LG} C_{LG} V_{iG})\, dz - d\Phi_G - \left(\frac{\rho_G A_G g_L \overline{V}_G}{g_c}\right) dz \qquad (4)$$

The momentum equation can be applied to the case of two-phase annular or stratified flow with no liquid entrained in the gas phase by adding Equation (3) to the analogous equation for the liquid phase, neglecting surface tension (7), and noting therefore that $\tau_{LG} = -\tau_{GL}$, and also $C_{LG} = C_{GL}$, $V_{iG} = V_{iL} = V_i$:

$$\frac{d(\overline{W_L V_L}) + d(\overline{W_G V_G})}{g_c} + A\, dp + (\rho_L A_L + \rho_G A_G) \frac{g_L}{g_c} dz = (\tau_{WG} C_{WG} + \tau_{LW} C_{WL})\, dz \qquad (5)$$

A similar equation can be obtained from the mechanical energy equation. Dividing Equation (4) by $\overline{V}_G$, assuming no net mass transfer to the phase, then adding to this the corresponding equation for the liquid phase, subject to the boundary conditions listed above, one obtains

$$\frac{d(\overbrace{W_L V_L}) + d(\overbrace{W_G V_G})}{g_c} + Adp + (\rho_L A_L + \rho_G A_G) \frac{g_L}{g_c} dz = -\left(\frac{d\,\Phi_G}{\overline{V}_G} + \frac{d\,\Phi_L}{\overline{V}_L}\right) + \tau_{LG} C_{LG} V_i \left(\frac{1}{\overline{V}_G} - \frac{1}{\overline{V}_L}\right) \qquad (6)$$

Comparison of Equations (5) and (6) indicates that the simple relationship between wall shearing stress and total viscous dissipation which exists in single-phase flow, $\tau_W C_W = -\dfrac{1}{\overline{V}} \dfrac{d\Phi}{dz}$, does not exist in two-phase flow. Not only are the individual dissipation terms weighted by the velocities of the individual phases, but an additional term is present representing the work done by one phase on the other across the gas-liquid interface, also weighted by the fluid velocities.

The mechanical energy equation as shown by Equation (4) for a gas phase can be added to the corresponding equation for a liquid phase with the elimination of the interfacial work term by noting that work done by the gas phase on the liquid phase is the negative of work done by the liquid phase on the gas phase (*4*):

$$\frac{d(\overline{W_G V_G^2}) + d(\overline{W_L V_L^2})}{2\ g_c} + (A_L\overline{V_L} + A_G\overline{V_G})\ dp + (\rho_L A_L\overline{V_L} + \rho_G A_G\overline{V_G})\ \frac{g_L}{g_c}\ dz = -\ d\ (\Phi_L + \Phi_G) \tag{7}$$

The result is an equation in which the viscous dissipation terms appear unencumbered with other variables.

In practice the momentum terms in Equation (5) and the kinetic energy terms in Equation (7) are usually negligible relative to other terms in the equations. The pressure gradient can then be related to $\mathcal{F}$ and to $d\Phi/dz$:

$$\mathcal{F} = A\frac{dp}{dz} + \frac{g_L}{g_c}\ (\rho_L A_L + \rho_G A_G) \tag{8}$$

$$-\frac{d\Phi}{dz} = (A_L\overline{V_L} + A_G\overline{V_G})\ \frac{dp}{dz} + \frac{g_L}{g_c}\ (\rho_L A_L\overline{V_L} + \rho_G A_G\overline{V_G}) \tag{9}$$

Either $\mathcal{F}$ or $d\Phi/dz$ can be related to the fluid and kinematic properties by a theoretical hydrodynamic analysis (*1*, *4*) or by dimensional analysis (*2*). It is important however that these two equations not be confused with one another. In Equation (9) the contribution of each phase to total dissipation is weighted by the mass flow rate of the phase. In Equation (8) the contribution of each phase to total wall shearing stress is weighted by the mass flow rate of each phase divided by the velocity of that phase. The manner in which the contributions of each phase are weighted in the mechanical energy equation, [Equation (7)] differs from that proposed by Isbin and Su (*6*). These authors make use of an incomplete mechanical energy equation in which work done across the interface is neglected. This equation is then weighted to obtain an equation similar in form to the momentum equation, which when compared with the momentum equation leads to an erroneous relationship between shearing stress and energy dissipation as can be seen by comparing Equations (5) and (6) above with those obtained by Isbin and Su. While it is correct to use either Equation (8) or (9) as a basis for two-phase flow pressure-drop correlations, interchange of $\mathcal{F}$ and $-\ d\Phi/dz$ in these equations will lead to incorrect results.

Equations (8) and (9) can be generalized to n phase flow to give

$$\mathcal{F} = A\frac{dp}{dz} + \frac{g_L}{g_c}\sum_{j=1}^{n} p_j A_j$$

$$-\frac{d\Phi}{dz} = \left(\sum_{j=1}^{n} A_j\overline{V}_j\right)\frac{dp}{dz} + \frac{g_L}{g_c}\left(\sum_{j=1}^{n} p_j\ A_j\ \overline{V}_j\right) \tag{10}$$

The expressions discussed in the previous paragraphs can be generalized to other flow regimes such as slug flow, bubble flow, and spray flow, all of which involve phase discontinuities, through the use of macroscopic momentum and energy balances.

NOTATION

A = area, L^2
C = length of interfacial perimeter, L
F = body force, F
$\mathcal{F}$ = wall shearing force per unit length = $\sum_{j=1}^{n} (\tau_{wj}C_{wj})$, F/L
g_c = reciprocal of proportionality constant in Newton's second law of motion, ML/FT^2
g_L = acceleration of gravity, L/T^2
p = pressure, F/L^2
t = time, T
V = point velocity, L/T
$\overline{V}$ = $\frac{\int V\,dA}{A}$, L/T
$\overline{WV}$ = $\int \rho V^2\,dA$, ML/T^2
$\overline{WV^2}$ = $\int \rho V^3\,dA$, ML^2/T^3
$\underbrace{WV}$ = $\overline{WV^2}/\overline{V}$, ML/T^2
z = length in direction of flow, L
Φ dissipation function = $\int (\overline{\overline{\tau}}\cdot\nabla)\cdot\mathbf{V}\ d$ (volume) FL/T
ρ = fluid density, M/L^3
$\underline{\underline{\tau}}$ = viscous stress tensor, F/L^2
τ = shearing stress, F/L^2

Subscripts

i = interface
G = gas
GL = gas-liquid interface
L = liquid
LG = liquid-gas interface

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INFORMATION RETRIEVAL

Key Words: Photolysis-I, Reactions-I, Photochemical-I, Chlorination-I, Vapor Phase-H, Free Radical-H, Reactions-H, Reaction Mechanics-H, Reaction Kinetics-H, Kinetics-H, Chloroform-A, Chlorinated Hydrocarbons-A, Halogenated Hydrocarbons-A, Chlorine-A, Carbon Tetrachloride-B, Chlorinated Hydrocarbons-B, Halogenated Hydrocarbons-B, Uranyl Sulfate-A, Sulfates (Inorganic)-A, Oxalic Acid-A, Acids (Carboxylic)-A, Light (Energy)-D, Turbulence-F, Reynolds Number-F, Intensity-F, Rates-G, Reactors-J, Chlorinators-J, Models-J.

Abstract: Data on the vapor-phase photochlorination of chloroform in flow reactors are interpreted on the basis of reaction mechanisms presented in the literature. In order to estimate the rates of light absorption it is also necessary to interpret the rate data obtained for the uranyl sulfate-oxalic acid reaction. The effects of flow regime and light intensity on the chlorination reaction rate are presented and explained.

Reference: Huff, J. E., and C. A. Walker, **A.I.Ch.E. Journal, 8,** No. 2, p. 193 (May, 1962).

Key Words: Reactions-H, Rates-H, Decomposition-I, Dissociation-I, Ethane-A, Ethylene-A, Hydrocarbons-A, Olefins-A, Vinyl Compounds-A, Hydrogen-B, Ethylene-B, Acetylene-B, Hydrocarbons-B, Olefins-B, Vinyl Compounds-B, Temperature-F, Dilution-F, Rates-G, Constants-G, Tubes-J, Shock-.

Abstract: The rates of decomposition of ethane and ethylene have been measured at temperatures between 1,250° and 2,800°K. and pressures between 0.25 and 2.0 atm. by exposing the gases to high temperatures for short intervals in a shock tube. Over-all decomposition rates were measured and compared with those predicted by extrapolation of low-temperature data. The effects of the addition of other gases in amounts up to 17% also were studied.

Reference: Miller, I. F., and S. W. Churchill, **A.I.Ch.E. Journal, 8,** No. 2, p. 201 (May, 1962).

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